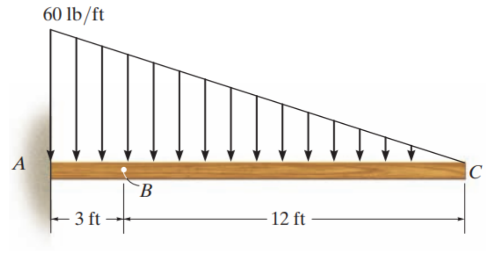
Problem 1-5

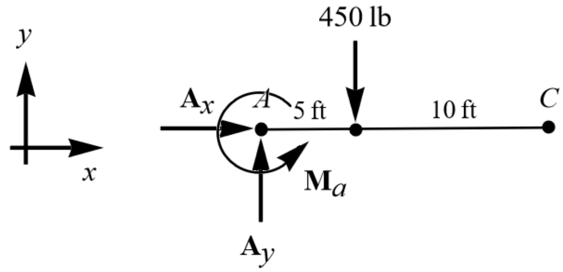
Determine the resultant internal loadings acting on the cross section at point B.



Prob. 1–5

Solution

The fixed support at A prevents horizontal motion, vertical motion, and rotational motion. As a result, there are three unknowns in the free-body diagram of the beam shown below.



The distributed force is treated as a single resultant force applied through the centroid of the triangle,

$$\frac{2}{3}(15 \text{ ft}) = 10 \text{ ft}$$

from point C, with a magnitude given by the area of the triangle,

$$\frac{1}{2}\left(60\ \frac{\text{lb}}{\text{ft}}\right)(15 \text{ ft}) = 450 \text{ lb}.$$

Use the equations of equilibrium to determine the reaction forces, taking the sum of the moments about point A.

$$\sum F_x = A_x = 0$$

$$\sum F_y = A_y - 450 = 0$$

$$\circlearrowleft^{+} \sum M_A = M_a - (450 \text{ lb})(5 \text{ ft}) = 0$$

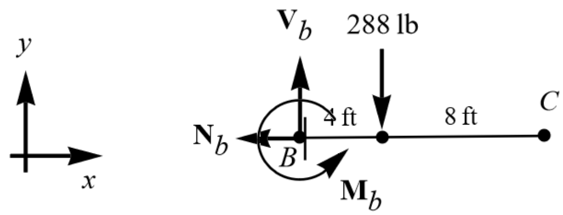
Solving this system of equations yields

$$A_x = 0$$

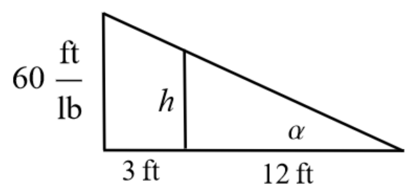
$$A_y = 450 \text{ lb}$$

$$M_a = 2250 \text{ lb} \cdot \text{ft}.$$

Now that the reactions at A are known, the resultant internal loadings acting on the cross section at B can be determined with the method of sections. Since it's simpler, choose the section of the beam from B to C.



This 288-lb force is the resultant force of the part of the distributed force from B to C. The height of the triangle at B is found using similar triangles.



$$\tan\alpha = \frac{60}{15} = \frac{h}{12} \quad \rightarrow \quad h = 48\ \frac{\text{lb}}{\text{ft}}$$

The force's magnitude is the area,

$$\frac{1}{2}\left(48\ \frac{\text{lb}}{\text{ft}}\right)(12\text{ ft}) = 288\text{ lb},$$

and the force passes through the centroid, which is

$$\frac{2}{3}(12\text{ ft}) = 8\text{ ft}$$

from C. Use the equations of equilibrium to find the internal loadings at B.

$$\sum F_x = -N_b = 0$$

$$\sum F_y = V_b - 288 = 0$$

$$\circlearrowleft^{+} \sum M_B = M_b - (288\text{ lb})(4\text{ ft}) = 0$$

Solving this system of equations yields

$$N_b = 0$$

$$V_b = 288\text{ lb}$$

$$M_b = 1152\text{ lb}\cdot\text{ft}.$$

The reactions at A actually weren't necessary.